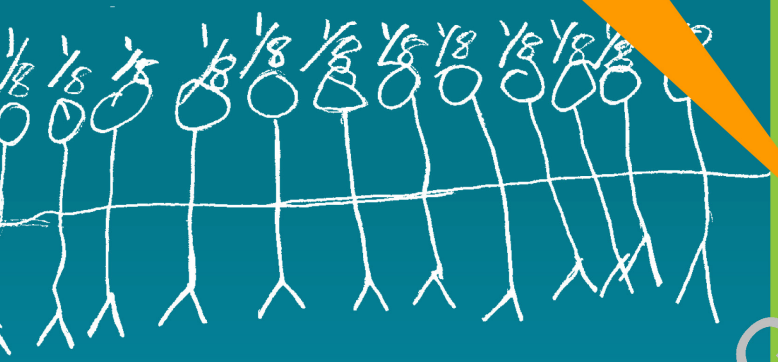
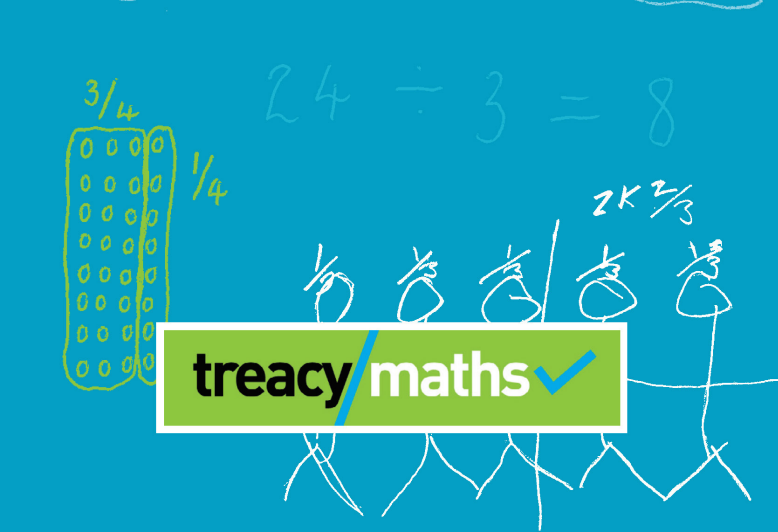
30 diagnostic tasks that teachers can use to identify students' understanding of various concepts of fractions.

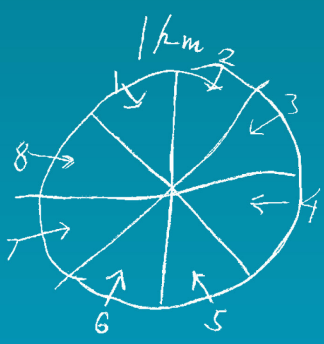
REVEALING WHAT STUDENTS THINK

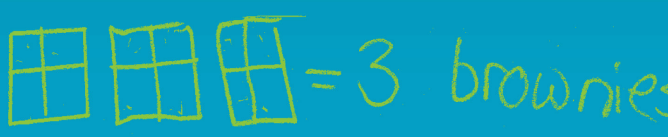
Diagnostic Tasks for Fractional Numbers

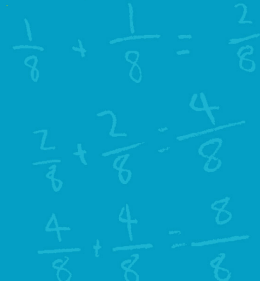
Text











treacy/maths

Task 13 Naming Fractions

Each task has a clearly defined purpose, as well as ideas for modifying the task based on student interest/need.

Purpose

The purpose of this task is to find out whether students can use written symbols to name fractions including $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ and $\frac{1}{6}$. Often students are able to label and identify fractions when given pre-drawn shapes that have been evenly partitioned and shaded but are not able to name the fractions when the shapes have uneven partitions.

This task is suitable for students aged nine to twelve years.

Adapting the Task to Accommodate Students' Background Experiences

This task could be adapted to be used as an interview. Photocopy the sheet onto a piece of coloured card and then cut out the pieces. Place each piece one at a time on top of the original diagram and ask children to name the fraction. If they can name the fraction, ask them to write it down. This will allow you to see whether they know the written symbol for the spoken fraction word.

Interpreting Students' Responses

Most middle primary students are familiar with pre-partitioned shapes where they are asked to name fractions. The unequal partitions in this task mean that students have to decide on the size of each partition in relation to the rest of the shape. In the rectangular shape, students may be able to name the half and quarter sections, as these are very familiar; however, they may struggle to work out the other parts of the shape. Some students will count the pieces and use this number as the denominator; others will think of fractions that are smaller than $\frac{1}{4}$ and so arrive at $\frac{1}{5}$ or $\frac{1}{6}$ for the section that is $\frac{1}{8}$, and then use $\frac{1}{8}$ for the section that is $\frac{1}{16}$. Other students who correctly work out the size of the pieces will do so by using a process of continuous halving.

Students may be able to name the quarters within the circle but have difficulty in working out the size of the sixths. Children are usually able to find eighths and sixteenths more easily than sixths, as the first two fractions can be found by a continuous halving process, whereas sixths cannot.

Teachers who want to know more about the fractions concept that is being assessed will find this section particularly helpful.

Task adapted from Clarke, Mitchell & Roche, 2005.

Task 13 Naming Fractions

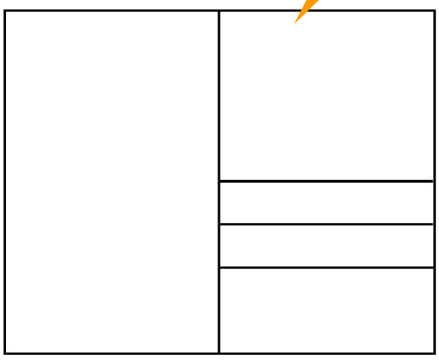
NAME:

A student record sheet is included with every task. Teachers may photocopy these for use in the classroom.

Part One

How many different fractions can you see in the diagram

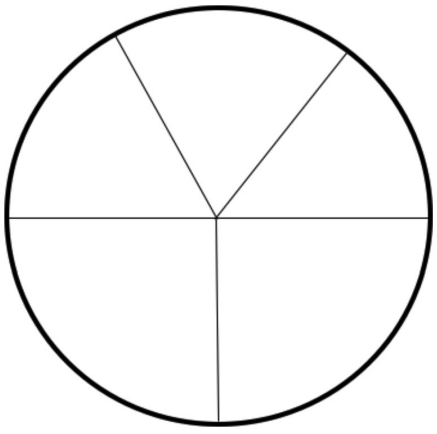
Label as many as you can.



Part Two

How many different fractions can you see in the diagram below?

Label as many as you can.



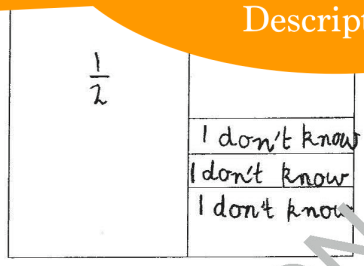
Task 13 Naming Fractions

Every task also includes examples of student work which provides teachers with a range of possible outcomes / Performance Descriptors.

Performance Descriptor A
Lauren, Year 5

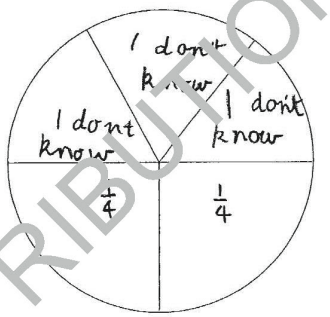
In Part One, Lauren (right) is able to name and use the written symbols for the half and quarter sections but is unable to use these as the basis for working out half of $\frac{1}{4}$ and half of $\frac{1}{8}$.

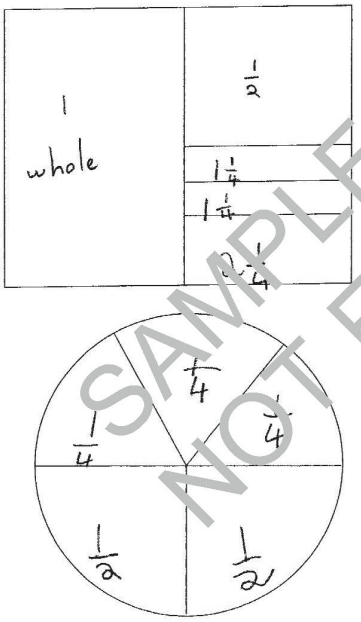
In Part Two, Lauren can see the quarter sections but is not familiar enough with other fractions to work out the remaining sections as $\frac{1}{6}$.



Performance Descriptor B
Rachel, Year 4

In Part One, Rachel (below) sees the half section as the whole and then compares the other sections to this whole. From this perspective, it is appropriate to name the top right section as a half.





She has then visualised cutting the remaining half into four pieces and so has named the smaller middle sections as quarters, though she has written them as $1\frac{1}{4}$. Students often write quarters like this when they are beginning to learn to use the written symbols. They see the written symbol $\frac{1}{4}$ as a 'quarter' and place a 1 in front of this to say how many quarters they have. They see it as 'one, quarter' not as 'one *and* one quarter'. Rachel thinks the lower right section is $\frac{2}{4}$ and has written this as 'two, quarters'.

In Part Two, Rachel sees the two lower sections as half of the lower half of the diagram. She knows that the top sections are smaller than the halves and so has used the fraction that she is familiar with – that is, quarters – to name these sections.

Planning for Learning

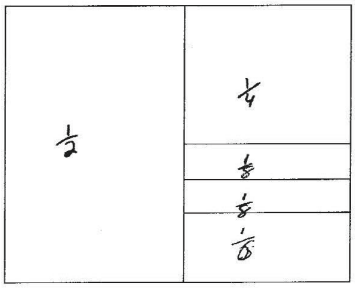
Lauren and Rachel seem to be familiar with halves and quarters but not with other fraction names or symbols. Activities that extend this range by using a variety of different types of objects would be helpful. These should begin with eighths and sixteenths, as students can find these easily by working out half of a quarter and half of an eighth, and then move on to thirds, followed by sixths (half of a third) and fifths. Activities that require students to create the partitions for themselves are far more likely to help them … different fractions. Rachel's attention in each activity shou… symbols is written. ✯

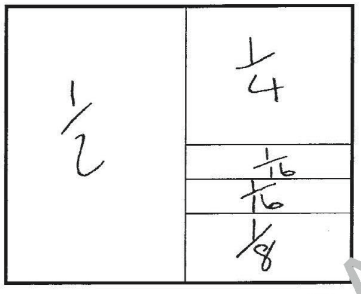
Suggested learning activities, based on the Performance Descriptors, are also provided. Teachers can use these to plan for instruction, and to guide them in their selection of activities from their current curriculum resources.

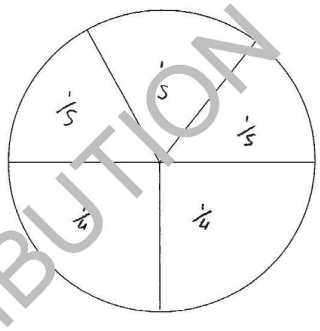
Performance Descriptor C
Alex, Year 5

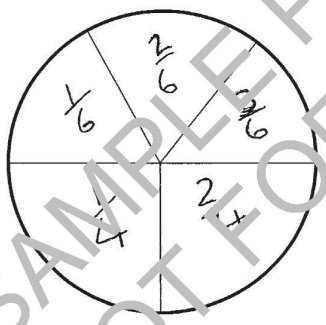
In Part One, Alex (right) has been able to correctly identify the half and quarter sections. He is aware that the lower right section is smaller than the quarter and thinks that a sixth is smaller than a quarter. This leaves the two middle sections: he knows that eighths are smaller than sixths and so uses this to label the remaining sections as eighths.

In Part Two, Alex has correctly labelled the quarters and has labelled the remaining sections as fifths. He counted the number of sections to find five in all and concluded that the sections must therefore be fifths.









Performance Descriptor D
Kasiana, Year 6

Kasiana (left) has correctly labelled all of the sections in the diagram in Part One; however, in Part Two she has counted the sections as she moved from left to right, and so has written $\frac{1}{4}$, $\frac{2}{4}$, $\frac{1}{6}$, $\frac{2}{6}$ and $\frac{3}{6}$. This is not incorrect, as she has identified the size of the sections in relation to each other.

Planning for Learning

Alex knows about fifths, sixths and eighths but has not been able to correctly attach these labels to the different fractional parts. He would benefit from similar activities to those suggested for Lauren and Rachel, with the focus on the size of the pieces in relation to each other. His knowledge could be extended by getting him to create 'jigsaws' of various shapes. Start with a collection of shapes, all the same, and cut them into halves, one into quarters, one into eighths and one into sixteenths, and then label the pieces appropriately. Use these to create the shape using different combinations of fractions.

Kasiana shows that she can name these common fractions but it is not clear whether she can use this understanding to name equivalent fractions. See Task 19, Three Quarters of a Pie.

Each performance descriptor has a suggested plan for learning to guide teachers in their instructional planning.

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NAVIGATION GRID - PERFORMANCE TASKS FOR FRACTIONS: Where to Start?

Tasks taken from Revealing What Students Think: Diagnostic Tasks for Fractional Numbers[1]

Major Content Focus	**Reason with Shapes and their attributes**	**Understanding Parts and Wholes**	**Equivalence**	**Partitioning Fractions**	**Compare and Order Fractions**	**Counting by Fractional Amounts**	**Adding Fractions**	**Multiplying and Dividing with Fractions**	**Locating Fractions on a Number Line**	**Understand Decimal Notation for Fractions**	**Understand and Use Ratios**	**Your Notes**
Task Name												
1. Licorice	■											
2. Zoo Animals		■	■									
3. Find a Half		■	■									
4. Feeding the Rabbits		■	■									
5. Sharing Bananas		■										
6. Walking to School				■								
7. Which Is Bigger?					■							
8. Cooking at Home					■							
9. Fruit Bowl		■										
10. Sharing Lollies		■										
11. Counting by Fractions			■			■						
12. Broken Eggs			■				■					

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Major Content Focus	Reason with Shapes and their attributes	Understanding Parts and Wholes	Equivalence	Partitioning Fractions	Compare and Order Fractions	Counting by Fractional Amounts	Adding Fractions	Multiplying and Dividing with Fractions	Locating Fractions on a Number Line	Understand Decimal Notation for Fractions	Understand and Use Ratios	Your Notes
Task Name												
13. Naming Fractions		■										
14. Running a Race			■			■						
15. After School			■									
16. Who Knows Best?			■									
17. Cookies			■									
18. Toy Cars				■				■				
19. Three Quarters of a Pie			■	■								
20. Jumping Competition					■							
21. How Long Is the Snake?					■				■			
22. Number Lines									■			
23. Circle the Bigger					■							
24. Pocket Money					■							
25. Party Food				■				■				

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Major Content Focus	Reason with Shapes and their attributes	Understanding Parts and Wholes	Equivalence	Partitioning Fractions	Compare and Order Fractions	Counting by Fractional Amounts	Adding Fractions	Multiplying and Dividing with Fractions	Locating Fractions on a Number Line	Understand Decimal Notation for Fractions	Understand and Use Ratios	Your Notes
Task Name												
26. Brownies - Yum!								■				
27. Doing Homework Together					■					■		
28. Visit to the Zoo											■	
29. Making Lemonade											■	
30. 'More' Game			■		■					■		

[1]Students will also get a great deal of practice from the majority of these tasks in using fractional language and associated symbolic forms.

PERFORMANCE TASKS FOR FRACTIONS – Where to Start?

Tasks taken from Revealing What Students Think: Diagnostic Tasks for Fractional Numbers

Task Name	Purpose	Major Content Focus[1]	Thinking flexibly about fractions as[2]…	Related & Supporting CCSS
1. Licorice	Part One: Whether students understand that a half means one out of two parts, where the two parts are of equal quantity. Part Two: Whether students have generalized their understanding of halves and use the word 'half' to mean one piece out of two equal-sized pieces.	• Reason with shapes and their attributes	• Part of a continuous whole	• 1.GA.3 • 2.GA.3 • 3.NF.A.1 • 3.NF.A.3.A
2. Zoo Animals	Whether students have generalized the idea that a half means one out of any two equal-sized parts.	• Understanding parts and wholes • Equivalence	• Part of a collection	• 3.NF.A.1 • 3.NF.A.3 • 3.NF.A.3.A • 3.NF.A.3.B
3. Find a Half	Whether students have generalized the idea that a half means one out of any two parts, where the two parts are of equal quantity, and that the half can be half of one object or half of a collection of objects.	• Understanding parts and wholes • Equivalence	• Part of a continuous whole and • Part of a collection	• 1.GA.3 • 2.GA.3 • 3.NF.A.1 • 3.NF.A.3 • 3.NF.A.3.A • 3.NF.A.3.B • 4.NF.A.1
4. Feeding the Rabbits	Whether students can use continuous halving to make four equal-sized portions, use up the whole item, and name the resulting portion as one quarter.	• Understanding parts and wholes • Equivalence	• Part of a continuous whole	• 1.GA.3 • 2.GA.3 • 3.NF.A.1 • 3.NF.A.3 • 3.NF.A.3.A

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Task Name	Purpose	Major Content Focus[1]	Thinking flexibly about fractions as[2]…	Related & Supporting CCSS
5. Sharing Bananas	Whether students are able to share three items between two people, by giving each person one item each and then cutting the remaining item into two equal-sized portions.	• Understanding parts and wholes	• Division	• 3.NF.A.1
6. Walking to School	Whether students are able to use continuous halving to work out a fractional amount and to use the associated symbolic form	• Partitioning fractions • Using unit fractions (*a fraction with 1 as its numerator, e.g. ⅕*)	• A quantity or measure[3]	• 4.NF.B.3.A • 4.NF.B.3.B • 4.NF.B.3.D • 5.NF.A.2
7. Which Is Bigger?	Whether students know that 1/4 is smaller than 1/3 because the more portions something is split into, the smaller each portion is.	• Compare and order fractions	• Numbers	• 3.NF.A.1 • 3.NF.A.3.D
8. Cooking at Home	Whether students know that 1/4 is smaller than 1/3 because the more portions something is split into, the smaller each portion is, and… Whether they are able to consider the size of the numerator and the denominator when they are comparing 2/3 and 3/4.	• Compare and order fractions	• A quantity or measure[3]	• 3.NF.A.1 • 3.NF.A.3.D • 4.NF.A.2
9. Fruit Bowl	Whether students are able to think of a collection of apples and pears as one whole collection of fruit and name the fraction of apples as two fifths.	• Understanding parts and wholes	• Part of a collection	• 3.NF.A.1 • 4.NF.B.3 • 4.NF.B.3.B • 4.NF.B.3.D • 5.NF.A.2
10. Sharing Lollies	Whether children can use the fractions a third, two thirds and a quarter to describe parts of a whole when the whole is a collection of items.	• Understanding parts and wholes	• Part of a collection	• 3.NF.A.1 • 4.NF.B.3.B • 4.NF.B.3.D • 5.NF.A.2

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Task Name	Purpose	Major Content Focus[1]	Thinking flexibly about fractions as[2]…	Related & Supporting CCSS
11. Counting by Fractions	Whether children are able to count by fractional amounts.	• Counting by fractional amounts • Equivalence	• Numbers	• 3.NF.A.2 • 3.NF.A.2.A • 3.NF.A.2.B
12. Broken Eggs	Whether students can use written symbols to name less common fractions within collections of items, particularly 5/12, 7/12 and 3/12. It will also show whether they can add the fractions 5/12 and 3/12 in a practical situation.	• Adding fractions • Equivalence	• Part of a collection	• 3.NF.A.1 • 4.NF.B.3.A • 4.NF.B.3.D • 5.NF.A.2
13. Naming Fractions	Whether students can use written symbols to name fractions including 1/2, 1/4, 1/8, 1/16 and 1/6.	• Understanding parts and wholes	• Part of a continuous whole	• 3.NF.A.1
14. Running a Race	Whether students are able to count by eighths and thirds beyond 'one' to solve a problem. It may also show evidence of whether students can calculate with fractions.	• Counting by fractional amounts • Equivalence	• Numbers	• 3.NF.A.1 • 3.NF.A.3 • 5.NF.B.6
15. After School	Whether students have an understanding of simple equivalent fractions like 3/4 = 6/8 = 9/12.	• Equivalence	• Numbers	• 3.NF.A.3.A • 3.NF.A.3.B • 4.NF.A.1 • 4.NF.A.2 • 5.NF.A.1
16. Who Knows Best?	Whether students have an understanding of fractions as quantities or whether they have more of a rote understanding that does not allow them to think of the fraction 3/4 in this situation in practical terms. It may also show evidence of whether students can calculate with fractions.	• Equivalence	• A quantity or measure[3]	• 3.NF.A.1 • 3.NF.A.3.A • 3.NF.A.3.B • 4.NF.A.1 • 4.NF.A.2 • 5.NF.A.1

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Task Name	Purpose	Major Content Focus[1]	Thinking flexibly about fractions as[2]…	Related & Supporting CCSS
17. Cookies	Whether students are able to name fractions of a collection, eight twenty-fourths and six twenty-fourths, and some fractions that are equivalent to these.	• Equivalence	• Parts of a collection	• 3.NF.A.3.A • 3.NF.A.3.B • 3.NF.A.3.D • 4.NF.A.1 • 4.NF.A.2
18. Toy Cars	Whether students are able to solve a problem when given the number of objects for 3/4 of a collection as 24, and have to find out how many objects are in the whole collection.	• Partitioning Fractions • Multiplying and dividing fractions	• Parts of a collection	• 4.NF.B.3.B • 4.NF.B.4.A • 5.NF.B.3
19. Three Quarters of a Pie	Whether students are able to work out equivalent fractions to find a quantity of pie – that is, one sixth of three quarters of the pie. It may also show whether students can calculate with fractions.	• Equivalence • Partitioning Fractions	• Division	• 4.NF.A.2 • 4.NF.B.3 • 5.NF.A.1 • 5.NF.A.2 • 5.NF.B.6 • 5.NF.B.7.A • 6.NSA.1
20. Jumping Competition	Whether students can compare the size of fractional numbers 1/3 and 1/4 on a number line and/or count in units of a third and a quarter.	• Compare and order fractions	• Numbers	• 3.NF.A.2 • 3.NF.A.2.A • 3.NF.A.2.B
21. How Long Is the Snake?	Part One: Whether students are able to read fractions on a number line, and to use this to combine fractions and to compare the size of fractions. Part Two: Whether students can use a number line to accurately draw creatures of particular fractional lengths (5/8, 2 1/4) and to compare the two fractions 2/3 and 3/4.	• Compare and order fractions • Locating fractions on a number line	• Numbers	• 3.NF.A.2 • 3.NF.A.2.A • 3.NF.A.2.B • 5.NF.A.2 • 5.NF.A.2

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Task Name	Purpose	Major Content Focus[1]	Thinking flexibly about fractions as[2]…	Related & Supporting CCSS
22. Number Lines	Whether students are able to think of fractions as numbers rather than just as quantities and to locate them on a number line.	• Locating fractions on a number line	• Numbers	• 3.NF.A.2 • 3.NF.A.2.A • 3.NF.A.2.B
23. Circle the Bigger	Whether students are able to compare fraction numbers using benchmark fractions – for example, 4/9 is less than a half, or 7/8 is close to one.	• Compare and order fractions	• Numbers	• 3.NF.A.3 • 3.NF.A.3.D • 4.NF.A.2 • 5.NF.A.2
24. Pocket Money	Whether students consider the size of the whole when asked to compare fractions.	• Compare and order fractions	• A quantity or measure[3]	• 3.NF.A.1 • 3.NF.A.3.D • 4.NF.A.2
25. Party Food	Whether students are able to work out shares in a situation where there are more shares required than objects – for example, more children than pieces of garlic bread.	• Partitioning Fractions • Multiplying and dividing fractions	• Division	• 5.NF.B.3 • 5.NF.B.5.B • 5.NF.B.6 • 5.NF.B.7.A • 5.NF.B.7.B • 5.NF.B.7.C
26. Brownies - Yum!	Whether students are able to see the relationship between fractions and division.	• Multiplying and dividing fractions	• Division	• 5.NF.B.3 • 5.NF.B.5.B • 5.NF.B.6 • 5.NF.B.7.A • 5.NF.B.7.B • 5.NF.B.7.C
27. Doing Homework Together	Part One: Whether students have a sense of the size of the fraction 1/8 and of the decimal numbers 0.8 and 0.125 and know that 1/8 is a different name for 0.125 Part Two: Whether students have a sense of the size of the improper fraction 4/3	• Understand decimal notation for fractions • Compare and order fractions	• Related to decimals and percentages	• 4.NF.C.5 • 4.NF.C.6 • 4.NF.C.7

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Task Name	Purpose	Major Content Focus[1]	Thinking flexibly about fractions as[2]…	Related & Supporting CCSS
28. Visit to the Zoo	Whether students are able to use fractions to express ratio relationships and use these to work out a larger quantity in a situation involving a collection.	• Understand and use ratios	• Ratios	• 6.RPA.1 • 6.RPA.2 • 6.RPA.3 • 6.RPA.3.A
29. Making Lemonade	Whether students are able to use fractions as ratios to work out a larger amount of liquid from a smaller one.	• Understand and use ratios	• Ratios	• 6.RPA.1 • 6.RPA.2 • 6.RPA.3 • 6.RPA.3.A • 6.RPA.3.D
30. 'More' Game	Whether students have an understanding of the relationships between fractions, percentages and decimals. You will also get more information about students' understanding of equivalent fractions.	• Understand decimal notation for fractions • Compare and order fractions • Equivalence	• Related to decimals and percentages	• 4.NF.C.5 • 4.NF.C.6 • 4.NF.C.7

[1]Students will also get a great deal of practice from the majority of these tasks in using fractional language and associated symbolic forms.

[2]Fractions can be interpreted in many different ways depending on the context of the problem and students need practice with all these interpretations in order to learn to think flexibly about fractions. For the purposes of the tasks in *Revealing What Students Think: Diagnostic Tasks for Fractional Numbers*, the following are used: Fractions as A) part of a continuous whole; B) part of a collection; C) numbers; D) division; E) a quantity or measure; F) ratios; G) related to decimals and percentages

[3]Fractions can also be used to describe other categories of quantity – or measurements - such as time, distance, area, volume, mass, weight, money…